# Writing the Augmented Matrix of a System of Equations

A matrix can serve as a device for representing and solving a system of equations. To express a system in matrix form, extract the coefficients of the variables and the constants, and these become the entries of the matrix. A vertical line is used to separate the coefficient entries from the constants, essentially replacing the equal signs. When a system is written in this form, it is called an **augmented matrix**.

\*Note: Before writing a system of equations as an augmented matrix it is necessary to put all the variable terms on one side of the equations and the constant term on the other.

Given a system of equations, write an augmented matrix.

1) Write the coefficients of the terms as the numbers down the first column.

2) Write the coefficients of the terms as the numbers down the second column.

3) If there are -terms, write the coefficients as the numbers down the third column.

4) Draw a vertical line and write the constants to the right of the line.

Example: Write the augmented matrix for the given system of equations.

# Writing a System of Equations from an Augmented Matrix

We can use augmented matrices to help us solve systems of equations because they simplify operations when the systems are not encumbered by the variable. However, it is important to be able to move back and forth between formats.

Example: Find the system of equations represented by the augmented matrix.

# Performing Row Operations on a Matrix

Various **row operations** can be performed on a matrix, such as addition, multiplication by a constant, and interchanging rows. Performing row operations on a matrix is the method we use to solve a system of equations. In order to solve the system of equations, the goal is to convert the matrix to **row-echelon form**, in which there are ones down the main diagonal from the upper left corner to the lower right corner, and zeros in every position below the main diagonal as shown below:

To solve a system of equations we can perform the following row operations to convert the coefficient matrix to row-echelon form and do back-substitution to find the solution.

1) Interchange rows. (Notation: )

2) Multiply a row by a constant. (Notation: )

3) Add the product of a row multiplied by a constant to another row. (Notation: )

To obtain a matrix in row-echelon form for finding solutions, we use Gaussian elimination.

The **Gaussian elimination** method refers to a strategy used to obtain the row-echelon form of a matrix. The goal is to write matrix with the number as the entry down the main diagonal and have all zeros below.

The first step of the Gaussian strategy includes obtaining a as the first entry, so that row may be used to alter the rows below.

Given an augmented matrix, perform row operations to achieve row-echelon form.

1) The first equation should have a leading coefficient of . Interchange rows or multiply by a constant, if necessary.

2) Use row operations to obtain zeros down the first column below the first entry of .

3) Use row operations to obtain a in row , column .

4) Use row operations to obtain zeros down column , below the entry of .

5) Use row operations to obtain a in row , column .

6) Continue this process for all rows until there is a in every entry down the main diagonal and there are only zeros below.

7) If any rows contain all zeros, place them at the bottom.

Examples

1. Perform row operations on the given matrix to obtain row-echelon form.
2. Solve the given system by Gaussian elimination.

# Solving a System of Linear Equations Using Matrices

We have seen how to write a system of equations with an augmented matrix, and then how to use row operations and back-substitution to obtain row-echelon form. Now, we will take row-echelon form a step farther to solve a by system of linear equations – eliminate all but one variable using row operations and then back-substitute to solve for the other variables.

Examples: Solve the following systems using matrices.

## Solving a System Using a Calculator

Given a system of equations, solve with matrices using a calculator.

1) Save the augmented matrix as a matrix variable.

2) Use the **ref(** function in the calculator, calling up each matrix variable as needed.

Examples

1. Solve the system of equations using a calculator.
2. For each of the following, set up the augmented matrix that describes the situation, and solve for the desired solution.
   * + 1. At a competing cupcake store, worth of cupcakes are sold daily. The chocolate cupcakes cost each and the red velvet cupcakes cost each. If the total number of cupcakes sold per day is , how many of each flavor are sold per day?
       2. A bag of mixed nuts contains cashews, pistachios, and almonds. Originally there were nuts in the bag. of the almonds, of the cashews, and of the pistachios were eaten, and now there are nuts left in the bag. Originally, there were 100 more cashews than almonds. Figure out how many of each type of nut was in the bag to begin with.